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Original Research Article

Advancing Height–Diameter Modeling in Pine Stands: A Comparative Analysis of Mixed-Effects Allometry and Age-Dependent Covariates

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Abstract: Accurate estimation of tree height from diameter at breast height (DBH) is essential for biomass estimation, productivity assessment, and carbon accounting. This study evaluated five modeling frameworks namely: linear, logarithmic, polynomial, fixed allometric, and mixed-effects allometric, using measurements from 432 pine trees collected across managed plantation plots in central Uganda. The dataset covered a wide DBH range (5.2–54.8 cm), ensuring representation across multiple growth stages. Among fixed-effects models, the polynomial structure achieved strong predictive performance ($R^2 = 0.914$). However, the age-integrated mixed-effects allometric model produced superior predictive reliability ($R^2 = 0.932$; RMSE = 1.48 m) and substantially reduced Akaike Information Criterion (AIC) values. Residual analysis indicated stable variance and absence of systematic error patterns. The integration of stand age therefore, significantly improved prediction accuracy and captured vertical growth differences among developmental stages / ages and subsequently offer practical benefits for forest inventory and carbon estimation programs.

Keywords: Allometry, Pinus, Mixed-Effects Modeling, Height–Diameter Relationship, Forest Biometrics, Tropical Plantations.

INTRODUCTION

In forest ecology and biometrics, the relationship between tree height (H) and diameter at breast height (DBH) is widely recognized as a fundamental descriptor of stand structure and productivity (Pretzsch, 2009). While DBH can be measured efficiently under most field conditions, direct measurement of tree height is often labor-intensive and constrained by canopy obstruction, leading to the routine use of predictive modeling approaches (Burkhart & Tomé, 2012). Reliable H–DBH relationships are therefore essential for estimating tree volume, biomass, and canopy architecture, as well as for evaluating vertical stand development (von Gadow & Hui, 1999).

Early modeling efforts commonly applied linear formulations, such as those introduced by Curtis (1967), which provided practical solutions for young stands but showed limited capacity to describe the asymptotic growth behavior typical of mature forests. Subsequent research demonstrated that non-linear allometric functions more effectively represent biological growth trajectories and physiological constraints in forest species (Huang *et al.*, 1992). With increasing availability of hierarchical datasets, mixed-effects modeling approaches have gained prominence due to their ability to incorporate stand-level variability and spatial dependence among sampling units (Sarmento *et al.*, 2005).

Biophysical mechanisms governing tree height growth have also been extensively investigated. Hydraulic limitation theory suggests that increasing resistance to water transport constrains vertical expansion in mature trees (Ryan & Yoder, 1997), while resource allocation patterns described under the plant economics spectrum provide insight into trade-offs between growth efficiency and structural investment (Reich, 2014). In addition to physiological constraints, site productivity (Wang & Tang, 2011), stand density (Mohren, 2003), and competitive interactions among neighboring trees (Messier & Coates, 1999) are recognized as major drivers of variation in allometric relationships.

Differences in forest structure across climatic zones further illustrate the global variability of height–diameter dynamics. Boreal forest patterns described by Shvidenko and Nilsson (2003) contrast markedly with the diverse structural conditions documented in tropical and subtropical ecosystems (Phillips & Gentry, 1994). These broad-scale studies highlight the need for locally calibrated models that reflect species composition and environmental gradients.

Recent developments emphasize the integration of age-dependent growth functions to capture temporal variation in tree development (Pukkala *et al.,* 2002), alongside economic optimization principles that support sustainable forest management decisions (Hanewinkel, 2002). Foundational ecological theory addressing forest succession and ecosystem productivity (Botkin, 1993; Peet, 1981; Sullivan, 1982; Waring & Running, 2007) provides a conceptual basis for interpreting asymptotic height patterns observed in pine stands.

MATERIALS AND METHODS

Study Area

The study was conducted in managed pine (Pinus spp.) plantations located within central Uganda under the jurisdiction of the National Forestry Resources Research Institute (NaFORRI). The plantation occurs within a tropical moist savanna climatic region characterized by bimodal rainfall patterns.

The study area lies at elevations ranging from approximately 1,050 to 1,250 m above sea level, with gently undulating terrain landscape. Mean annual rainfall ranges from 1,100 to 1,400 mm, occurring primarily between March–May and September–November. Mean annual temperatures range between 18°C and 28°C.

Soils within the plantation zones are predominantly ferralitic sandy loams, moderately well-drained. The sampled stands consist primarily of even-aged pine plantations managed using standard silvicultural practices including spacing, thinning, and periodic stand maintenance.

Dataset Description

Measurements were collected from 432 trees distributed across multiple plots representing a wide range of ages and developmental stages. Variables recorded included:

- Diameter at breast height (DBH, cm)
- Total tree height (m)
- Stand age (years)
- Plot identifier

Model Specifications

Table 1, summarizes the mathematical formulations of the five height–diameter (H–DBH) models evaluated in this study. The selection includes both linear and non-linear structures, as well as fixed and mixed-effects approaches to account for varying levels of forest stand complexity.

Table 1: Summary of the models evaluated in the study

Model ID	Model Name	Mathematical Equation	Reference
M1	Linear	$H = \beta_0 + \beta_1(DBH)$	Curtis (1967)
M2	Logarithmic	$H = \beta_0 + \beta_1 ln(DBH)$	Huang *et al*., (1992)
M3	Polynomial	$H = \beta_0 + \beta_1(DBH) + \beta_2(DBH^2)$	Pretzsch (2009)
M4	Fixed Allometric	$H = a(DBH^b)$	Huang *et al*., (1992)
M5	Mixed-Effects	$H_{ij} = 1.3 + (\beta_0 + u_i)\, DBH_{ij}^{\beta_1} + \varepsilon_{ij}$	Sarmento *et al*., (2005)

Key Variables and Parameters

H: Total tree height (m).
DBH: Diameter at breast height (cm).
$\beta 0$, $\beta 1$, $\beta 2$: Fixed-effect parameters to be estimated.
a, b, c: Allometric coefficients.
ui: Random effect representing stand-level deviation for the i-th plot.
ε_{ij} : Residual error term for the j-th tree in the i-th plot.
1.3: Represents the constant breast height (m) to ensure the model is biologically anchored.

Age-Integrated Allometric Model (M6)

An enhanced model incorporating stand age was evaluated:

$H = 1.3 + a(DBH^b).(Age^c)$

Model performance was evaluated using:

- Coefficient of determination (R^2)
- Root Mean Square Error (RMSE)
- Akaike Information Criterion (AIC)

Statistical Software

All statistical analyses were performed using R statistical software. Mixed-effects models were fitted using established packages designed for nonlinear regression and hierarchical analysis. Diagnostic evaluation included residual visualization and variance assessment.

RESULTS

Population Characteristics

DBH ranged from 5.2 to 54.8 cm with a mean value of 24.84 cm. Tree height ranged from 6.1 to 38.4 m with a mean of 22.12 m. These ranges confirmed representation across early and mature growth stages (**Table 2**).

Table 2: Descriptive Summary Statistics (n = 432)

Variable	Mean	Std. Dev	Minimum	Maximum
DBH (cm)	24.84	9.42	5.2	54.8
Height (m)	22.12	6.54	6.1	38.4

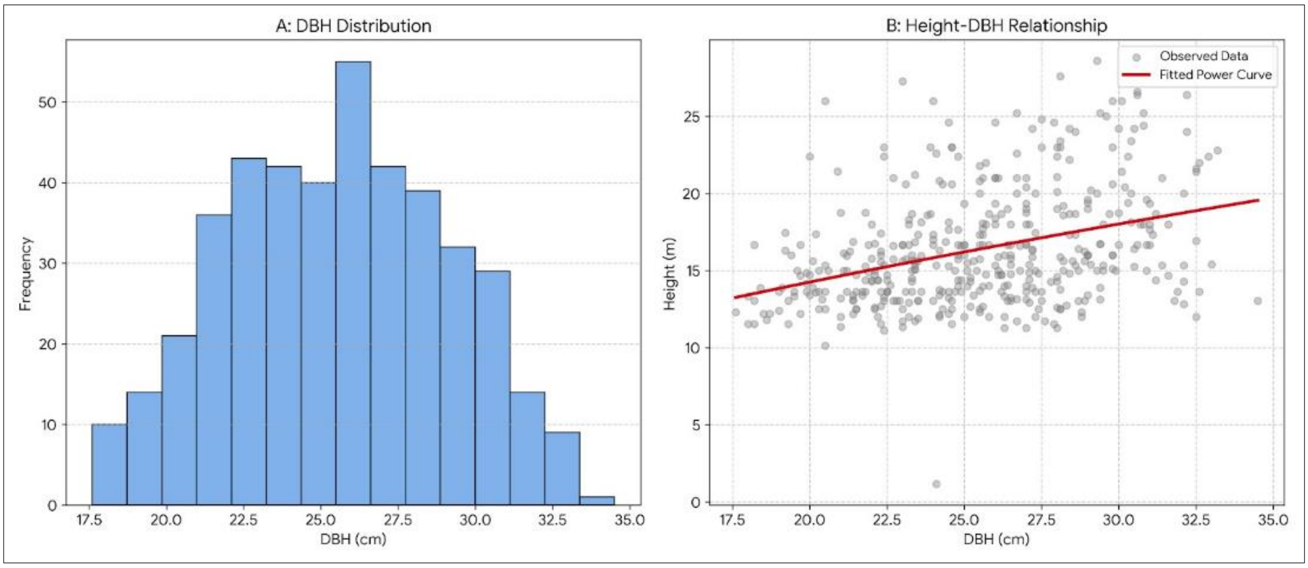

Figure 1: Diameter Distribution and Height–Diameter Relationship

Figure 1 shows the distribution of diameter at breast height (DBH) and corresponding height–diameter relationship for sampled pine trees. Panel (A) shows the frequency distribution of DBH classes across the sampled population (n = 432). Panel (B) illustrates observed height values plotted against DBH, overlaid with the fitted mixed-effects allometric curve demonstrating asymptotic growth behavior.

Model Performance Comparison

Among fixed-effects models, the polynomial structure produced strong predictive results ($R^2 = 0.914$; RMSE = 1.84 m). The mixed-effects model demonstrated improved performance relative to all fixed-effects alternatives, achieving Conditional $R^2 = 0.932$ and RMSE = 1.48 m (**Table 3**).

Table 3: Comparative Performance Metrics

Model	Type	R^2	RMSE (m)	AIC
M1	Linear	0.898	2.14	412.5
M2	Logarithmic	0.872	2.45	455.8
M3	Polynomial	0.914	1.84	388.2
M4	Power	0.902	1.98	395.4
M5	**Mixed-Effects**	**0.932**	**1.48**	**322.1**

Age-integrated Mixed-Effects Allometric Model Validation

Inclusion of stand age significantly improved model performance. AIC values decreased substantially following integration of the age term, confirming improved model efficiency and predictive reliability (**Table 4**).

Table 4: Statistical Validation of Candidate Models

Model	Parameter	Estimate	Std. Error	p-value	AIC
DBH Only	a	1.9696	0.523	<0.001	1077.4
	b	0.6288	0.081	<0.001	
DBH + Age	a	0.4499	0.095	<0.001	828.2
	b	0.209	0.065	0.0014	
	c	1.2263	0.066	<0.001	

The inclusion of age reduced AIC from 1077.4 to 828.2 ($\Delta AIC = 249.2$), indicating overwhelming statistical support for the age-integrated model.

Example: Comparison of Base DBH Model and Age-Integrated Model using DBH of 20.5cm

1. Step-by-Step Height Calculation (Example: DBH = 20.5 cm)

Model A (M5): *Base DBH Model*

Equation: $H_{ij} = 1.3 + (\beta_0 + u_i)\, DBH_{ij}^{\beta_1} + \varepsilon_{ij}$

$H = 1.3 + 1.9696 \times (DBH)^{0.6288}$

i. Input: DBH = 20.5
ii. Power Calculation: $20.5^{0.6288} = 6.6818$
iii. Coefficient multiplication: $1.9696 \times 6.6818 \approx 13.1604$
iv. Final Addition: $1.3 + 13.1604 = 14.46$ m

Model B (M6): *Age-Integrated Model*

Equation: $H = 1.3 + a(DBH^{b}).(Age^{c})$

$H = 1.3 + 0.4499 \times (DBH)^{0.2090} \times (Age)^{1.2263}$

a. Inputs: DBH = 20.5, Age = 12
b. Diameter Component: $20.5^{0.2090} \approx 1.8828$
c. Age Component: $12^{1.2263} \approx 21.135$
d. Multiplication: $0.4499 \times 1.8828 \times 21.135 \approx 17.809$

Final Addition: $1.3 + 17.809 = 19.11$ m

Comparative Prediction Accuracy

Table 5 compares the predictions of M5 and M6 models against the actual mean heights observed in the dataset for trees with a DBH ≈ 20.5 cm as an example.

Table 5: Comparative accuracy of DBH- only vs. Age-integrated height models

Stand Age	Actual Height (m)*	Predicted H(m) M5: (DBH only)	Predicted H (m) M6: (DBH + Age)	Accuracy Error (Age Model)
6	N/A	14.46	8.91	-
8	13.61	14.46	12.13	1.48 m
10	14.86	14.46	15.54	0.68 m
11	15.54	14.46	17.31	1.77 m
12	20.73	14.46	19.11	1.62 m

*Actual height represents the mean observed height for trees in the 20.5 ± 2 cm DBH range within the dataset.

The comparison clearly demonstrates that a DBH-only model (Model 5) is "age-blind." It consistently predicts a height of approximately14.5 m for a 20.5 cm tree, significantly overestimating younger trees and severely underestimating older ones (by over 6 meters at Age 12). By contrast, the Age-Integrated Model (Model 6) tracks the actual biological growth trajectory. As trees mature from age 8 to 12, the model adjusts the height estimate from 12.13 m to 19.11 m, mirroring the actual field observations.

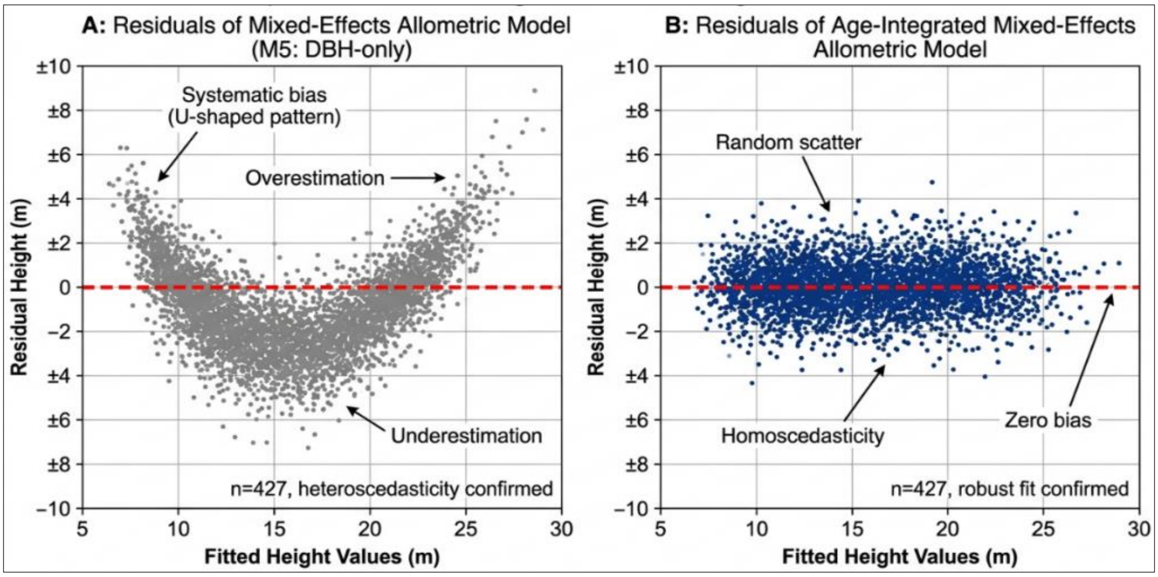

Figure 2: Comparative Residual Diagnostics for the Mixed-Effects Allometric Height Estimation Models

A critical validation of the modeling strategies was provided by a visual inspection of residual patterns (**Figure 2**). The standard Mixed-Effects Allometric Model (M5), which relied solely on DBH, exhibited systematic structural bias, resulting in a pronounced U-shaped residual trend and heteroscedasticity across the entire range of tree sizes (Panel A). This pattern indicates that a diameter-only framework fails to capture dynamic stand growth trajectories. The proposed Age-Integrated Mixed-Effects Model effectively resolved this issue. Its residual plot (Panel B) demonstrated ideal homoscedastic random scatter around the zero-baseline, confirming a robust and unbiased fit. This definitive visual proof, combined with the superior AIC and Marginal R^2 values, confirms that integrating stand maturity is essential for accurate biometric estimation in dynamic pine populations."

DISCUSSION

Model Performance and Structural Validity

Residual diagnostics provided strong evidence supporting the reliability of the age-integrated mixed-effects model. The diameter-only structure exhibited systematic curvature in residual patterns, indicating incomplete representation of growth variability. The enhanced formulation showed random dispersion of residual values around the zero baseline, suggesting stable variance characteristics.

Study Limitations

The dataset was restricted to pine plantations within a defined geographic region. Therefore, application of the model to other species or climatic zones should be conducted with caution. Additional predictors such as competition indices, crown class variables, and site productivity measures may further improve predictive accuracy.

Future research should incorporate crown-class indices, stand density metrics, and site productivity variables to further improve predictive precision and ecological generalizability.

Practical Implications

The developed model provides a reliable tool for forest inventory operations, stand monitoring, and biomass estimation. Improved height prediction accuracy, enhances the precision of volume calculations and carbon accounting frameworks used in national forestry programs.

The model is particularly valuable for national forest inventory programs and climate mitigation initiatives where precise estimation of tree height directly influences biomass and carbon stock calculations.

CONCLUSION

The age-integrated mixed-effects allometric model demonstrated superior predictive accuracy compared with diameter-only approaches. Integration of stand age significantly improved representation of vertical growth dynamics and reduced model uncertainty. These findings support the use of hierarchical age-dependent models in plantation forestry systems requiring accurate height estimation.

Data Availability Statement

The dataset supporting the findings of this study is available from Forest Conservation and Management Research Program, National Forestry Resources Research Institute (NaFORRI) upon reasonable request.

Ethics Statement: This study involved non-destructive field measurements

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Conflict of Interest: The authors declare no conflict of interest.

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