East African Scholars Journal of Education, Humanities and Literature
Abbreviated Key Title: East African Scholars J Edu Humanit Lit
ISSN: 2617-443X (Print) & ISSN: 2617-7250 (Online)
Published By East African Scholars Publisher, Kenya
Volume-9 | Issue-11 | Nov- 2026 | DOI: https://doi.org/10.36349/easjehl.2026.v09i11.004

OPEN ACCESS

Review Article

Thinking Framework and Transformation Strategies of the Segment Cutting and Extension Method in Geometric Reasoning

Xiaorong Zhang[1]*
[1]School of Economics and Management, Jiangsu Maritime Institute, Nanjing, 211170, China

Article History
Received: 06.09.2026
Accepted: 29.10.2026
Published: 11.11.2026

Journal homepage:
https://www.easpublisher.com

Quick Response Code

Abstract: As a classic auxiliary line construction approach, the segment cutting and extension method is widely adopted in geometric reasoning to convert problems about the quantitative relation of discrete line segments into proof tasks of congruent figures. This paper discusses the inherent logical rules of this method, sorts out its general thinking workflow, and proposes a taxonomy of transformation strategies indexed by the geometric transformation that the construction performs rather than by the appearance of the figure. Beyond the simple classification of problem types, this study further explores the cognitive barriers that learners usually encounter when applying the method, and compares this approach with other common auxiliary line techniques in geometric reasoning. Different from many existing discussions which only focus on case demonstration, this paper emphasizes the logical essence of geometric transformation rather than routine problem-solving skills. It also analyzes how this method reflects the core idea of the axiomatic geometry system, and provides corresponding suggestions for geometric reasoning guidance. The analysis shows that the value of segment cutting and extension lies not only in proving equal or multiple relations of line segments, but also in training learners to establish flexible association among angles, sides and graphic structures. This research can offer reference for the design of geometric reasoning activities and the exploration of transformation-oriented geometric thinking.

Keywords: Geometric Reasoning; Segment Cutting and Extension; Auxiliary Line; Geometric Transformation; Congruence Reasoning.

Copyright © 2026 The Author(s): This is an open-access article distributed under the terms of the Creative Commons Attribution **4.0 International License (CC BY-NC 4.0)** which permits unrestricted use, distribution, and reproduction in any medium for non-commercial use provided the original author and source are credited.

1. INTRODUCTION

Geometric reasoning occupies an essential position in the whole system of spatial and logical thinking training. From a historical perspective, elementary geometry has long been regarded as a natural carrier for cultivating deductive proof habits, because its propositions can be organized into a complete chain from axioms to conclusions [1, 2]. In geometric proof tasks, many propositions directly describe the quantitative relationship among multiple line segments, which cannot be verified by using the known properties of triangles or polygons in a straightforward way. Under such circumstances, constructing auxiliary lines becomes the most common path to bridge the gap between known conditions and target conclusions. Among all auxiliary line strategies, the segment cutting and extension method has distinctive characteristics. It operates by intercepting a segment of equal length on a longer line or extending a shorter segment to form an equal-length structure, so as to create a pair of congruent figures and transfer equal angles and equal segments between different regions of the graph.

A large number of existing discussions about this method mostly stay at the level of problem classification and case explanation. Relevant materials usually list typical graphic modes and corresponding operation steps, but seldom discuss the internal cognitive logic behind the operation [3]. Many learners can remember the fixed operation routines for familiar graph structures, yet fail to transfer this thinking mode when the graphic layout changes slightly. This phenomenon indicates that the mastery of this geometric method is not merely a memory of fixed problem-solving templates. It relies on the understanding of geometric invariance under transformation. Research on proof and proving also suggests that meaningful reasoning requires not only the acquisition of technical operations but also the construction of a convincing narrative that connects known conditions with the target statement [4-6].

***Corresponding Author:** Xiaorong Zhang
School of Economics and Management, Jiangsu Maritime Institute, Nanjing, 211170, China

In terms of theoretical research, geometric transformation has long been regarded as the core thread connecting elementary geometry knowledge. Translation, rotation and reflection all maintain the congruence of figures [7, 8]. The segment cutting and extension method is essentially a manual construction that simulates the effect of rotation or reflection transformation on local parts of a graph. It does not rely on advanced geometric tools, but realizes the reorganization of graphic elements within the boundary of basic congruence axioms. This feature makes it suitable for the cultivation of basic deductive reasoning. Moreover, recent studies on dynamic geometry environments emphasize that learners' spatial imagination can be effectively activated when figures are allowed to be moved and recombined, which is exactly the mental activity that auxiliary line construction requires [9, 10].

The primary research purposes of this paper are as follows. First, clarify the core connotation and theoretical foundation of the segment cutting and extension method, and distinguish it from other auxiliary line construction methods. Second, sort out a universal thinking framework that learners can follow when they try to use this method. Third, construct a taxonomy of transformation strategies indexed by the geometric transformation performed by the construction rather than by the observable configuration of the figure, and supplement the analysis of learners' typical cognitive obstacles. Fourth, discuss the guiding value of this method for geometric reasoning training. The research method adopted in this paper is theoretical analysis combined with typical case interpretation. All cases are described by text narration without mathematical formulas or symbolic expressions. By decomposing the reasoning chain of each scenario, this paper reveals how the transformation idea runs through the whole proof process. It is expected that such an explicit description of the reasoning chain can help both teachers and learners observe the hidden logic behind auxiliary line construction [11].

2. Core Connotation and Theoretical Basis of the Segment Cutting and Extension Method

2.1 Core Connotation

Segment cutting and extension includes two basic operation directions. Cutting means selecting a longer segment in the graph, intercepting a sub-segment which equals one of the known shorter segments, and then connecting relevant points to form a new triangle. Extension means prolonging a shorter segment outward, so that the extended whole segment is equal to a given longer segment, and then constructing a matching triangle for congruence reasoning. These two operations share the same logical goal: to decentralize scattered equal segment conditions and assemble them into a complete congruent structure.

It is necessary to emphasize that segment cutting and extension is not a universal method applicable to all segment relation problems. Its applicable premise is that the graph contains implicit equal angle conditions or angle bisector structures, and the target conclusion is an additive or multiplicative relation of line segments. If there is no available equal angle after auxiliary line construction, the operation of cutting or extending segments will not bring effective progress to the proof. This is an important boundary that many learners ignore. They tend to copy the operation mechanically whenever they see segment relation conclusions, without judging whether the angle conditions can support congruent reasoning. The core idea hidden behind the operation is transfer. After constructing congruent figures, equal line segments and equal angles can be transferred from one triangle to another. In this way, the original scattered conditions converge to the same graphic unit, and the complex segment relation can be transformed into the comparison of sides inside a single triangle.

2.2 Theoretical Basis from Geometric Axiom System

The reasoning of segment cutting and extension is built on the congruence axioms of planar geometry. Congruence itself is a description of geometric invariance. When two figures are congruent, the corresponding sides and corresponding angles keep consistent regardless of the position difference in the plane [12]. Cutting and extending segments artificially creates a pair of triangles that satisfy congruence conditions, so the invariant property of congruence can be used for reasoning. From the perspective of geometric transformation, congruence is equivalent to the composite result of reflection, rotation and translation [7]. Most segment cutting and extension operations correspond to local rotation or reflection. For example, in the scene with an angle bisector, the two sides separated by the bisector have symmetrical properties. Intercepting equal segments on one side is equivalent to mapping the point on that side to the other side through axial symmetry. In this sense, this method is a concrete, operable embodiment of transformation thinking in elementary geometric reasoning.

The axiomatic system of elementary geometry also provides the logical boundary for this method. Every construction step must be justifiable by existing axioms and theorems; otherwise the proof chain will be broken. This requirement of traceability is precisely what distinguishes a genuine geometric proof from a trial-and-error exercise [1]. The segment cutting and extension method, when applied correctly, always keeps the reasoning chain traceable, because each newly formed triangle is supported by explicitly stated congruence conditions. This property makes it a reliable candidate for building the habit of formal deduction in geometric reasoning activities.

2.3 Cognitive Psychology Basis

From the perspective of cognitive psychology, geometric reasoning belongs to visual deductive thinking [13]. Learners need to complete multiple steps of mental representation: identify known conditions, perceive the overall structure of the graph, predict the change brought by auxiliary lines, and verify whether new relations can be derived. Novice reasoners usually treat the graph as a static picture, while experienced reasoners can regard partial segments and angles as movable elements that can be reorganized [14]. The segment cutting and extension method helps learners break static graphic perception. The operation of intercepting or extending segments forces learners to imagine the recombination of graphic elements. This process promotes the transformation from perceptual visual observation to logical deductive reasoning.

The concept of representational systems offers a useful explanation for why some learners can transfer this method across problems while others cannot [15]. When a learner stores a geometric scene as a visual template coupled with a fixed sequence of operations, the representation is tightly bound to a specific layout and easily fails after any change of position. When the learner instead stores the reasoning as a relation among quantities that remain invariant under transformation, the representation becomes more abstract and more transferable. Training oriented to transformation thinking aims exactly at promoting this shift from surface visual representation to deeper structural representation [8, 9]. Such a shift is considered a key step in the development of geometric reasoning ability.

3. General Thinking Framework for Reasoning with the Segment Cutting and Extension Method

This paper sorts out a three-stage thinking framework, which can guide learners to carry out reasoning step by step instead of trying auxiliary lines blindly. The whole framework is descriptive and procedural, and it can be adapted to different geometric scenes. It is designed as a routine of attention, so that learners examine the conditions, plan the construction, and check the outcome in a fixed order [11].

3.1 Stage One: Condition and Target Decoding

The first stage is condition and target decoding. Reasoners first distinguish the known information in the proposition, focusing on equal angles, angle bisector, isosceles structures and equal side marks. At the same time, they clarify the target conclusion, confirming whether the conclusion expresses an additive relation of two segments equal to the third segment or a multiple relation between segments. This stage judges whether the problem is potentially suitable for segment cutting and extension. If the conclusion only asks for the size of an angle or the parallel relation of straight lines, this method is usually not the preferred choice. Explicitly marking the known conditions and the target conclusion at this stage helps to prevent the common mistake of trying auxiliary lines before fully understanding the proposition.

3.2 Stage Two: Scheme Selection and Auxiliary Line Design

The second stage is scheme selection and auxiliary line design. After confirming that the method is applicable, reasoners need to decide whether to adopt cutting or extension. The selection principle depends on the distribution of known equal angles. If there is a relatively long segment in the graph and the nearby angles have symmetric characteristics, the cutting scheme is usually preferred. If the existing short segments are distributed on one side of a fixed vertex and the angle condition is convenient for outward construction, the extension scheme is more appropriate. At this step, reasoners need to predict which new equal angles will appear after drawing auxiliary lines, and judge whether the predicted angles can form congruence matching. This predictive step is the most demanding part of the whole framework, because it requires imagining the recombined figure before the auxiliary line is actually drawn.

3.3 Stage Three: Congruence Derivation and Conclusion Feedback

The third stage is congruence derivation and conclusion feedback. After constructing the auxiliary line and forming a new triangle, reasoners list the matching conditions for congruent reasoning. Once congruence is confirmed, the corresponding side and angle relations can be transferred. Finally, reasoners check whether the transferred segment relation can support the target conclusion. If the reasoning chain is blocked at this stage, it is necessary to return to the second stage, adjust the position of interception or change between cutting and extension schemes. This cyclic framework highlights the predictive and verificatory nature of geometric reasoning. It avoids the common misunderstanding that segment cutting and extension is a single fixed operation.

3.4 Why the Framework Is Cyclic and Transferable

The three stages form a cycle rather than a one-way sequence. In actual reasoning, the first attempt frequently fails at the third stage, and the reasoner has to return to the second stage to adjust the construction. A cyclic routine is preferable because it treats an unsuccessful construction as valuable information instead of a dead end. Furthermore, the framework is transferable because it does not depend on any particular graph. The same order of decoding, designing and verifying can be applied to angle relation problems or area relation problems, which strengthens the learner's general reasoning habits rather than isolated skills [11, 6].

4. A Transformation-Operator Taxonomy and Its Reasoning Instantiations

The preceding section set out a three-stage framework that tells the reasoner when to examine a proposition, how to plan a construction, and how to verify the result. What the framework does not yet provide is a principled basis for choosing among possible transformations, and this is precisely the question that most existing treatments answer by enumeration. Available discussions of the method typically list a series of problem types, attach an operation routine to each type, and leave the choice to recognition: if the figure contains a bisector, apply the bisector routine; if it contains an isosceles triangle, apply the isosceles routine. The enumeration is serviceable inside the classroom, but it says nothing about why a particular routine works, and it offers no guidance when a figure belongs to none of the listed types. This section therefore abandons enumeration. Strategies are classified instead by the geometric transformation that the construction performs, by the invariant that the transformation transports, and by the source from which the closing angle is drawn. Four operators emerge from this reading, and they accommodate the ten routine operations that conventional treatments present as separate techniques: three modes of adjacency, three provenances of the mirroring axis, two modes of rotation, and two modes of calibration. The resulting taxonomy is more compact than the enumeration, it is predictive rather than recognitional, and, as the final subsection shows, it is able to state when the method should be given up.

4.1 Why a Figure-Type Classification Cannot Carry the Reasoning

Three costs follow from indexing a classification by the appearance of the figure [3]. The first is a failure of compression. The angle bisector scene, the isosceles scene, the double-angle scene and the half-angle scene appear as four separate entries in the conventional list, yet all four are executions of one transformation: a triangle is reflected across an axis, or turned about a pivot, until it coincides with another triangle. A learner who is given four entries has to maintain four routines for one idea, and the number of stored routines grows faster than the conceptual content they carry. The second is a failure of discrimination. Cutting and extending are conventionally presented as two halves of a single technique, so the learner is never told what separates a case that calls for moving an endpoint along a line from a case that calls for reflecting a triangle across an axis. When the first attempt fails, the learner has no vocabulary for saying what went wrong. The third is a failure of coverage. The conventional list is indexed by symmetry, and a proposition that asserts a sum relation in a figure offering no bisector, no isosceles triangle and no parallel pair has no entry at all, although such propositions are routinely solved by prolonging one of the shorter segments along its own line [11].

All three costs share one root. A classification indexed by appearance stores every strategy in the form of a visual template, and this is exactly the representational form that section 1 identified as the source of non-transfer and that section 5.1 will describe as template dependence [15]. The transformation itself supplies an alternative index, because the transformation remains the same object while the appearance of the figure changes. Two consequences follow for instruction. Strategies can be taught as families rather than as a list, and the question a learner puts to an unfamiliar figure becomes a question about transformations rather than a question about resemblance.

4.2 Two Governing Principles: Channel Separation and Angular Supply

Two principles organise the taxonomy, and both are consequences of what a cutting or extending operation can and cannot do. The first is channel separation. A cutting or extending operation relocates points along straight lines. It changes the metric arrangement of the figure, but it cannot produce a new angular magnitude, because no step of the operation fixes an angle: intercepting a segment on a ray, or prolonging a segment beyond its endpoint, adds collinear points and nothing more. Every angle that appears after the construction is therefore either copied from an angle already present, or implied by the collinearity of the new points, in which case it appears only as part of a supplementary pair. The operation acts on the length channel, while the congruence that it closes is always paid for out of the angle channel [7, 8]. The practical consequence is severe and simple: before drawing anything, the reasoner must be able to name the angle that will close the congruence and to state where that angle comes from. If the angle cannot be named in advance, the construction is a guess, and the third stage of the framework will fail.

The second principle is angular supply. The supply of a configuration is the stock of angle equalities and angle relations that a congruence criterion can consume, and its provenance is of three kinds. Given supply consists of what the conditions state explicitly: a marked pair of equal angles, an angle bisector, or the equal base angles of an isosceles triangle. Derived supply consists of equalities that follow from other stated relations, such as the alternate interior angles supplied by a pair of parallel lines, or the supplementary relation along a straight line. Manufactured supply consists of equalities that do not exist until something is done to the figure, as when a larger angle is bisected in order to obtain two angles equal to a smaller given one [14]. Section 2.1 stated the applicability premise of the method informally, as the requirement that the graph contain an implicit equal-angle condition; the notion of supply turns that premise into bookkeeping. The count is short. A congruence needs one pair of sides from the length channel, one angle from the supply, and one further

matching element, which may be a side, an angle, or a shared element such as a common side or a common vertex. Admissibility is then the requirement that the supply remaining after the planned construction be at least what the chosen criterion consumes.

The two principles yield a third, operational one. The difference between conditions and conclusion can be described as a gap, and that gap is of three kinds. A positional gap occurs when the segments whose relation is asserted are of the right magnitude but lie apart or in different directions, so that no comparison between them is possible in the figure as drawn. A magnitude gap occurs when the asserted relation is multiplicative, so that one segment must be resized before any comparison becomes meaningful. A channel gap occurs when the conditions speak about angles while the conclusion speaks about segments. Each gap admits one operator as its natural instrument, and Table 1 arranges the operators accordingly.

Table 1: Taxonomy of the four transformation operators and the ten instantiation modes

Operator	Construction performed	Invariant transported	Supply required	Typical instantiations
I. Adjacency	An endpoint is moved along a straight line until two segments abut, so that their sum or difference becomes a single segment	Segment length	Given or derived; the angle at the shared vertex, with the supplementary pair at the weld obtained at no cost	Sum and difference relations in figures that offer no symmetry axis
II. Mirroring	A triangle is reflected across an axis that is given, implicit, or manufactured before use	The congruence class as a whole, with orientation reversed	One of three provenances: given, implicit, manufactured	Bisector scenes; isosceles and equilateral scenes; double-angle and half-angle scenes
III. Rotation	A triangle is turned about a common vertex until one equal side falls on the other	Distance and angle, with the pivot fixed	Given; the angle at the pivot, or the difference between two stated angles	Equal sides meeting at one vertex; a half angle inside a larger angle
IV. Calibration	A length is resized in a fixed ratio about a centre, the ratio being one at the boundary case	The ratio between corresponding elements	Given or derived, and consumed twice: a ratio pair together with an included angle	Multiplicative conclusions; propositions that supply a midpoint

4.3 Operator I: Adjacency and the Welding of Collinear Segments

Operator I is the simplest of the four and the one that the conventional list omits entirely. Its operation is the relocation of an endpoint along a straight line, so that two segments become collinear and share a vertex; the asserted sum, or the asserted difference, then appears as a single segment whose endpoints the figure already connects. Three modes are in use. In the intercept-and-weld mode, a longer segment is cut at an interior point placed so that the resulting part equals a given shorter segment, after which the new point is joined to a vertex of the existing structure. In the outward-extension mode, a shorter segment is prolonged beyond its endpoint until the prolongation equals the remaining part, and the new endpoint is joined in the same way. In the overlap-and-cancel mode, used for difference conclusions, one segment is laid over the other, the common part cancels, and the residual appears as the difference the proposition requires.

The supply requirement of this operator is the lowest of the four, and the reason is structural. The weld point lies on a straight line, so the supplementary relation at that point costs nothing: it is implied by collinearity and needs no separate justification. What must be supplied from outside is only the angle at the vertex where the relocated endpoint is joined. Adjacency is therefore the natural instrument for propositions whose conditions contain a single equal-angle pair and nothing else, and it is also the natural fallback when a more ambitious construction fails.

Its characteristic failure is informative. A learner who prolongs a segment along the wrong ray produces an overlap rather than an abutment, and the angle that was meant to close the congruence is destroyed by the operation itself. This error is not a failure of memory; it is a failure to apply the channel principle, because the learner never checked which angle the weld would leave available. Learning to ask that question before drawing is the whole content of the operator.

4.4 Operator II: Mirroring and the Three Provenances of an Axis

The mirror operator reflects a triangle across an axis. Because a reflection preserves every distance and every angle while reversing orientation, the image triangle carries the entire congruence class of the original, and this is the largest invariant that any of the four operators transports. What varies among mirror

scenes is not the operation but the provenance of the axis, and on this basis three modes are distinguished [8].

When the axis is given, the figure already displays the symmetry. An angle bisector is the standard instance: the two angles separated by the bisector are equal, the two rays that form the angle are symmetrically placed, and intercepting an equal segment on one ray while joining its endpoint to a point on the other ray produces two triangles sharing the bisected angle, with one pair of equal sides and one common side. After congruence, the segment or angle hidden on one side of the bisector is transferred to the other side. An observation recorded in earlier discussions of this scene remains valid and can now be explained. Learners who have memorised that a bisector permits this method still fail when the bisector is not placed horizontally or vertically, because what they stored was the appearance of the axis rather than the fact of reflection. An axis is a relation, not a position, and recognising it should be a matter of locating two symmetric rays, not of matching a picture.

When the axis is implicit, the symmetry is present but not displayed. Isosceles and equilateral figures carry a natural axis through the apex, and the equal sides and equal base angles such figures contain are exactly the matching conditions a congruence criterion needs. The difficulty of this mode is that the isosceles structure is frequently not part of the original figure at all: it appears only after the construction has been made, so the reasoner has to predict a structure that is not yet visible. This mode therefore separates learners who reason from supply from learners who reason from sight.

When the axis is manufactured, the conditions state a ratio between angles rather than an equality, and no axis exists until one is drawn. A double-angle condition is the clearest instance. Nothing in the figure is symmetric, but bisecting the larger angle produces two angles equal to the smaller given one, and with this single step the scene becomes an instance of the given-axis mode. The structure usually described as a large angle containing a half angle belongs to the same mode, with one additional burden: the manufactured axis must at the same time satisfy a length condition at the shared vertex. The consequence deserves to be stated plainly, because it accounts for much of the difficulty attributed to these two scenes. They are not separate strategies; they are one operation performed with a deferred axis, and the deferral is what makes them hard. The auxiliary line that manufactures the axis leaves no trace in the final figure, since the conclusion reports a relation among segments; a learner who studies a finished solution therefore cannot recover the step from the answer, and the solution looks like an act of invention rather than a consequence of the conditions.

4.5 Operator III: Rotation, Its Certificate and Its Two Modes

The rotation operator turns a triangle about a common vertex until one of its sides falls on an equal side meeting at that vertex. Its invariant is distance together with angle, with the pivot fixed, so the image triangle is congruent to the original and the segment it carried is relocated elsewhere in the figure. The certificate that licenses the operation is short: a pair of equal sides sharing a vertex, together with the angle at that vertex.

Mirroring and rotation are easily confused, and the conventional list does not separate them, which is a further cost of indexing by appearance. The distinction is available to learners without any notation. A reflected image lies across a line from the original and its orientation is reversed, whereas a rotated image lies beside a pivot and its orientation is preserved. The diagnostic question is accordingly whether the image should appear across a line or around a point, and this single question replaces the attempt to recognise a named figure type.

Two modes are in use. The first serves structures in which equal sides already meet at a vertex: equilateral triangles and squares erected outward on the sides of a figure, and isosceles right triangles sharing a right-angle vertex. Here the certificate is displayed by the figure itself, and the operator is the cheapest of the four, because the supply is given and needs no manufacture. The second mode serves the half-angle structure. Earlier treatments of this scene noted that it resembles a rotation condition, and the taxonomy makes the resemblance exact: what the construction does is turn a part of the figure through the difference between two given angles, so that a segment occupying an awkward position is brought next to the segment it must be compared with. The rotation angle is the difficulty. It is never marked in the figure, because it is a difference between two stated angles rather than an angle appearing anywhere, so the reasoner must read an unmarked quantity as the parameter of a transformation. A second difficulty follows from the same source and is rarely mentioned. A rotation moves the image away from the original along an arc, so the figure looks more scattered after the construction than before it. Learners judge a construction by whether the diagram becomes tidier, and this operator systematically violates that expectation, which is why learners who are otherwise competent abandon it.

4.6 Operator IV: Calibration and the Continuum between Ratio and Congruence

The fourth operator resizes a length in a fixed ratio about a centre: cutting a segment at its midpoint resizes it by one half, prolonging a segment to twice its length resizes it by two. The invariant transported is the ratio between corresponding elements rather than any single length, and the resulting figure is similar to the original in that ratio. This operator stands on a continuum

with the other three rather than apart from them, because a ratio equal to one is precisely congruence; the four operators therefore differ in the invariant they preserve, not in the kind of reasoning they demand [8].

Two modes are distinguished by whether the ratio is stated by the conclusion or by the data. In the multiplicative mode, the conclusion asserts that one segment is twice or three times another, and the construction must be planned backwards from the required length: the reasoner fixes the target length before drawing, then chooses a centre and a ratio that produce it. In the midpoint mode, the data supply a midpoint and the available ratio is one half or two; the conclusion may still be additive, in which case the calibration first produces a segment that the adjacency or mirror operator then consumes.

The supply requirement of this operator is strictly higher than that of the other three, and this accounts for the difficulty that multiplicative conclusions are felt to have. A congruence consumes one side pair, one angle and one further element; a calibration has to satisfy a ratio pair and an included angle at the same time, so two matching relations are consumed instead of one. The difficulty of multiplicative problems is thus a difference in the accounting rather than a difference in kind, and this is the reason for treating them as a mode of calibration rather than as a separate problem type.

The same analysis clarifies a comparison that the conventional literature usually makes in operational terms. The median doubling method also extends a segment to twice its length, and it is often placed beside segment cutting and extending as a neighbouring technique. In operator terms the two are one first step applied to different targets: median doubling calibrates a median, which plays no role in the conclusion, and then completes the argument through a parallelogram; segment cutting and extending calibrates a segment that the conclusion itself names. The difference that matters for the learner is the difference in target, not in operation, and stating it this way becomes possible only after the classification has been moved away from appearance.

4.7 Gap Matching: A Selection Routine that Replaces Recall

The four operators and their modes can be assembled into a selection routine that replaces recall. The routine consists of three scans, performed in order, and it is short enough to be taught as a habit. The first scan reads the conclusion and asks which channel it speaks in. A conclusion asserting a sum or a difference of two segments in a figure without symmetry indicates adjacency. A conclusion asserting a ratio indicates calibration. A conclusion expressed in the angle channel indicates that the method is not applicable at all, because the operator acts on segments and has no target in an angle statement.

The second scan inventories the supply: which equalities are given, which can be derived from the stated relations, and which could be manufactured, and at what cost. The count required by the intended criterion is compared with the supply available, and the shortfall, if there is one, determines what must be manufactured before the length channel is touched. The third scan chooses the operator. If the gap is positional and the supply contains a usable angle but no symmetry, adjacency is indicated. If the supply consists of an equality of angles and an axis can be located or manufactured, mirroring is indicated. If equal sides meet at a vertex, rotation is indicated. If the gap is one of magnitude, calibration is indicated.

After the construction, closure is verified in the manner described in the third stage of the framework. What the taxonomy adds at this point is a repair rule that informal advice could not supply. When closure fails, the reasoner should change the supply rather than merely change the position of the point. Three repair moves are available: switch between cutting and extending while keeping the same supply, which consumes the same angle in a different way; manufacture a different axis, by bisecting an angle not chosen at first; or change the pivot, by selecting another vertex at which equal sides meet. Because each move is defined by the taxonomy, a failed attempt yields a specific next attempt rather than a repetition [11-16].

The same routine answers the question of variant layouts. When a figure is rotated, translated or partially redrawn, the operator performed by a construction is unchanged while the visual template is destroyed. The selection question should therefore be posed in operator terms, and this is the sense in which the taxonomy, rather than the list of types, is what transfers. A practical classroom rule follows from this: the learner should not ask what the figure looks like, but what is to be moved and what must be kept.

4.8 A Supply-Source Comparison with Other Auxiliary-Line Approaches

The taxonomy also reorganises the comparison between this method and other auxiliary-line approaches, because the comparison can now be made on one dimension instead of several. The question to ask of any auxiliary-line method is where its closing angle comes from, and what it costs to obtain it [7-10]. Drawing an altitude manufactures a supply of right angles, and this is why it can close a congruence in a figure that offers no equal angles at all; its cost is that it adds an element the proposition never mentioned and often destroys positional information the conclusion needs. Constructing the perpendicular bisector or the circumscribed circle imports supply from outside the local configuration, because the equal angles it uses follow from a theorem about circles rather than from the stated conditions; this supply is the most powerful and the most remote, since the reasoner must first notice four

points lying on one circle. Median doubling calibrates a length and then completes the argument through a parallelogram, which is the calibration operator followed by a half-turn about the midpoint, an instance of the rotation operator.

Segment cutting and extending differs from all of these in a single respect: it consumes only supply already present in the configuration, or supply that a single operation can manufacture from the stated relations. It imports no theorem and adds no element that the proposition does not already contain. This is the source both of its economy and of its limit. It is the cheapest instrument available when the local supply suffices, and it is unusable once the local supply is exhausted, a boundary that the next subsection examines.

4.9 Boundary Cases: Three Refusals the Taxonomy Can Make

A classification is useful only if it can also refuse. The taxonomy supports three refusals that the conventional enumeration, having no exclusion criterion, cannot make. The first concerns non-closure. If the conclusion is of a kind the length channel can state, but no local operation can manufacture the angle the congruence requires, the method is inadmissible, and the correct decision is to change the source of supply rather than to keep cutting and extending. This is the situation in which learners persist longest, because the enumeration taught them that failure means an insufficiently memorised routine, so they repeat the same operation at a slightly different position instead of abandoning the approach. A rule that permits abandonment is a substantive addition, not a concession.

The second refusal concerns channel mismatch. A conclusion that reports an angle magnitude, a parallel relation or an area relation gives the operator no target, since every operation in the length channel acts on segments. Earlier discussions of the method stated this as a matter of suitability; the taxonomy states it as a structural consequence, because an object cannot be transformed by an operator that does not act on it. The third refusal concerns chained relations. Adjacency welds two segments into one, so a conclusion relating three or more segments requires a chain of welds rather than a single one. Such chains are solvable, but their order is determined by the supply and not by the figure: each weld consumes an angle, so the weld that uses the scarcest angle must be placed where the supply is richest. The enumeration offers no principle for ordering a chain, which is why multi-segment conclusions are usually presented in textbooks as separate worked examples rather than as one reasoning pattern.

In sum, this section has replaced a list of problem types with a taxonomy of transformations. The unit of analysis is no longer the appearance of a figure but the operation performed on it, the invariant it transports, and the source of the angle that closes the reasoning. The gain is threefold: four operators replace six or more memorised types, the choice among them becomes a matter of accounting rather than of resemblance, and the boundary of the method becomes statable. The next section uses this vocabulary to describe the obstacles learners actually encounter.

5. Common Cognitive Barriers and Guiding Suggestions

In the practice of geometric reasoning, learners show several typical obstacles when using this method. Understanding these barriers is a necessary premise for designing effective guidance, because the teaching of an auxiliary line method is not complete if it only demonstrates successful examples [16].

5.1 Three Typical Cognitive Barriers

The first obstacle is template dependence. Learners only recognize several fixed graph styles summarized in exercise materials and cannot identify applicable conditions after the graph is rotated, translated or partially adjusted. The second obstacle is the separation between operation and logic. Learners can finish the auxiliary line drawing according to memory, but cannot state which congruence condition is supported and why this operation can help achieve the conclusion. The third obstacle is the inability to switch flexibly between cutting and extension. Once one scheme fails, they tend to give up reasoning instead of trying the other construction direction. All three barriers share a common root: the learner's representation of the method is stored as a rigid visual script rather than as a flexible relation among invariant quantities [15, 16].

5.2 Guiding Suggestions for Geometric Reasoning

Corresponding guiding suggestions can be put forward. First, more variant graphs should be provided in reasoning activities. The same conditional structure can be presented in different placement directions to weaken the dependence on static graphic templates [9]. Second, reasoners should be required to narrate the prediction before drawing auxiliary lines. They need to state which equal angles or equal sides they expect to obtain after construction. This forces the thinking to precede the drawing action and is consistent with the predictive nature of the three-stage framework [11]. Third, the internal consistency of cutting and extension should be emphasized. The two operations are equivalent transformation paths for the same geometric relation, not two isolated skill points. Finally, dynamic feedback should be encouraged: when a construction fails, the teacher can guide the learner to record what the failed attempt revealed, so that the next attempt becomes more targeted rather than a repetition of the same routine [10-17].

6. CONCLUSION AND OUTLOOK

The segment cutting and extension method is far more than a trick for geometric proof. Its essential value lies in carrying the transformation idea of

elementary geometry, using manual auxiliary line construction to reorganize graphic elements, and converting complex segment relation problems into congruence reasoning tasks [7, 1]. The three-stage thinking framework and the four-operator taxonomy of transformation strategies proposed in this paper provide a procedural reasoning path. Combined with the cognitive characteristics of geometric reasoning, this paper further analyzes the common obstacles of learners and compares this method with other auxiliary line approaches, supplementing the discussion beyond simple case demonstration.

This research still has limitations. The discussion in this paper stays within the scope of planar elementary geometry. It does not expand to curved geometry or spatial geometric scenes. Future exploration can focus on how to transplant the transformation idea of segment cutting and extension into spatial geometric reasoning, and design more continuous reasoning activities to examine the long-term effect of transformation thinking training [2]. In addition, quantitative research can be carried out to record the difference in reasoning performance between learners who receive transformation-oriented guidance and learners who only accept template-based skill training [6]. Such empirical evidence would provide stronger support for the theoretical claims made in this paper.

Geometric reasoning training ultimately aims at shaping the habit of logical deduction. Mastery of a single auxiliary line method is only a carrier. What deserves more attention is whether learners can form the awareness that geometric relations remain invariant under reasonable transformation. The cultivation of this awareness should be regarded as a long-term goal of geometry education, and the segment cutting and extension method offers a concrete and accessible entry point for realizing this goal [4-13].

REFERENCES

1. Hoffer, A. (1981). Geometry is more than proof. The Mathematics Teacher, 74(1), 11–18.
2. Jones, K., & Tzekaki, M. (2016). Research on the teaching and learning of geometry. In Á. Gutiérrez, G. C. Leder, & P. Boero (Eds.), The second handbook of research on the psychology of mathematics education: The journey continues (pp. 109–149). Sense Publishers.
3. Herbst, P. G. (2002). Establishing a custom of proving in American school geometry: Evolution of the two-column proof in the early twentieth century. Educational Studies in Mathematics, 49(3), 283–312.
4. Mariotti, M. A. (2006). Proof and proving in mathematics education. In Á. Gutiérrez & P. Boero (Eds.), Handbook of research on the psychology of mathematics education: Past, present and future (pp. 173–204). Sense Publishers.
5. Furinghetti, F., & Morselli, F. (2011). Beliefs and beyond: Hows and whys in the teaching of proof. ZDM Mathematics Education, 43(4), 587–599.
6. Stylianides, A. J. (2007). Proof and proving in school mathematics. Journal for Research in Mathematics Education, 38(3), 289–321.
7. Coxford, A. F., & Usiskin, Z. (1971). Geometry: A transformation approach. Laidlaw Bros.
8. Hollebrands, K. F. (2003). High school students' understandings of geometric transformations in the context of a technological environment. The Journal of Mathematical Behavior, 22(1), 55–72.
9. Sinclair, N., & Bruce, C. D. (2015). New opportunities in geometry education at the primary school. ZDM Mathematics Education, 47(3), 319–329.
10. Laborde, C. (2000). Dynamic geometry environments as a source of rich learning contexts for the complex activity of proving. Educational Studies in Mathematics, 44(1–2), 151–161.
11. Driscoll, M., Wing DiMatteo, R., Nikula, J., & Egan, M. (2007). Fostering geometric thinking: A guide for teachers, grades 5–10. Heinemann.
12. Edwards, L. D. (1991). Children's learning in a computer microworld for transformation geometry. Journal for Research in Mathematics Education, 22(2), 122–137.
13. Battista, M. T. (1990). Spatial visualization and gender differences in high school geometry. Journal for Research in Mathematics Education, 21(1), 47–60.
14. Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 420–464). Macmillan.
15. Goldin, G. A. (1998). Representational systems, learning, and problem solving in mathematics. The Journal of Mathematical Behavior, 17(2), 137–165.
16. Chazan, D. (1993). High school geometry students' justification for their views of empirical evidence and mathematical proof. Educational Studies in Mathematics, 24(4), 359–387.
17. De Villiers, M. (2004). Using dynamic geometry to expand mathematics teachers' understanding of proof. International Journal of Mathematical Education in Science and Technology, 35(5), 703–724.

Cite This Article: Zhang, X. (2026). Thinking Framework and Transformation Strategies of the Segment Cutting and Extension Method in Geometric Reasoning. *East African Scholars J Edu Humanit Lit, 9*(11), 660-668. https://doi.org/10.36349/easjehl.2026.v09i11.004