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Review Article

Teaching Research on Linear Algebra Based on Polya's "Three Principles of Learning"

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Abstract: This paper delves into the in-depth application of Polya's "Three Principles of Learning" within the realm of linear algebra teaching. By intricately integrating the principles of "actively learning", "learning by analogy and induction", and "reflecting and summarizing" into the teaching framework, it endeavors to not only boost students' learning enthusiasm and efficiency but also to foster their independent thinking, critical analysis, and complex problem - solving capabilities. Through a combination of real - world teaching cases, in - depth theoretical dissections, and empirical data analysis, the paper comprehensively demonstrates the far - reaching effectiveness of this teaching approach in promoting students' profound understanding and proficient mastery of linear algebra knowledge.

Keywords: Polya's "Three Principles of Learning", Linear Algebra, Teaching Research, Active Learning, Analogy and Induction, Reflection and Summary.

1. INTRODUCTION

Linear algebra, as a cornerstone course in mathematics, serves as an indispensable tool across a wide spectrum of disciplines, including physics, engineering, computer science, and data analytics. Its concepts underpin fundamental operations in quantum mechanics, circuit analysis, machine learning algorithms, and computer graphics. However, the abstract nature of its concepts, such as vectors, matrices, and linear transformations, and the complexity of its theoretical constructs often pose significant challenges to students. This has led educators to constantly seek innovative teaching methods to enhance the teaching quality of linear algebra and enable students to better internalize its knowledge.

Polya's "Three Principles of Learning" offers a revolutionary approach to teaching reform. These principles, deeply rooted in cognitive psychology and educational pedagogy, accentuate the significance of students' active engagement in the learning process, the strategic use of analogical and inductive reasoning for knowledge acquisition, and the crucial role of reflection and summary in consolidating learning [1-5]. Implementing these principles in linear algebra teaching can kindle students' intrinsic learning motivation, guide them towards acquiring efficient learning strategies, and ultimately enhance their overall academic and professional competencies.

2. Polya's "Three Principles of Learning"

2.1 The principle of "Actively Learning"

At the core of this principle lies the idea that students should be the architects of their own learning journey. Active learning is not merely about physical participation in class but also about mental engagement. It involves students actively formulating questions, critically analyzing problems, and collaboratively exploring solutions. In the context of cognitive load theory, active learning helps distribute the cognitive load effectively, preventing overloading of the working memory. For example, when students actively engage in problem - solving, they are more likely to chunk information, making it easier to transfer knowledge to long - term memory. This, in turn, stimulates their interest in learning, heightens their enthusiasm, and facilitates a more profound understanding and retention of knowledge.

2.2 The principle of "Learning by Analogy and Induction"

Analogy and induction are cognitive strategies that are fundamental to human learning. Analogy, as a form of reasoning, allows students to draw parallels between new and familiar knowledge domains. By

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identifying structural similarities, students can transfer existing knowledge to new situations, reducing the cognitive effort required to understand novel concepts. Induction, on the other hand, is the process of generalizing from specific instances to form general rules or principles. It is a key component of concept formation and theory building. In the context of linear algebra, these thinking methods can transform the learning of abstract concepts into a more accessible and intuitive process [3].

2.3 The principle of "Reflecting and Summarizing"

Reflection and summary are the meta - cognitive processes that enable students to monitor, evaluate, and adjust their learning. Reflection involves students looking back at their learning experiences, identifying what worked well and what needs improvement. This process is closely related to the concept of metacognition, which is the awareness and control of one's own thinking processes. Summary, on the other hand, is about organizing and integrating learned knowledge into a coherent framework. It helps students to form a mental map of the subject matter, facilitating better retrieval and application of knowledge [5-8].

3. Application of Polya's "Three Principles of Learning" in Linear Algebra Teaching

3.1 Implementing the Principle of "Actively Learning" In Linear Algebra Teaching

- **Problem - Based Learning**:
  - In linear algebra, the concept of matrix diagonalization can be taught through a problem - based approach. Teachers can present a real - world problem, such as analyzing the stability of a dynamic system in engineering. For instance, in a mechanical engineering scenario, the vibration of a multi - degree - of - freedom system can be modeled using matrices. By setting up the matrix equations representing the system, students are then guided to explore how matrix diagonalization can simplify the analysis of the system's behavior. As they work through the problem, they learn about eigenvalues, eigenvectors, and the process of diagonalizing a matrix. This not only makes the learning process more engaging but also shows students the practical significance of matrix diagonalization in real-world applications [7].
  - Another example is in the context of data analysis. Teachers can pose a problem of image compression using singular value decomposition (SVD), which is a matrix - based technique. Students are tasked with understanding how SVD can be used to represent an image matrix in a compressed form. They need to calculate the SVD of sample image matrices, observe how different levels of compression affect the image quality, and then discuss the trade - offs between compression ratio and image fidelity. This hands - on problem - solving approach allows students to actively participate in learning matrix operations related to SVD and understand its applications in data - intensive fields.

- **Group Cooperative Learning**:
  - When studying linear transformations, group cooperative learning can be highly effective. For example, in a group project, students can be assigned to explore the properties of linear transformations in the context of computer graphics. One group can focus on the rotation of 2D or 3D objects represented as vectors in a coordinate system. They need to derive the transformation matrices for different angles of rotation, implement these transformations in a simple graphics software (such as Python's Matplotlib or a basic graphics API), and then present their findings to the class. Another group can study the scaling and shearing transformations. Through group discussions, code sharing, and collaborative problem - solving, students can exchange ideas, learn from each other's perspectives, and jointly enhance their understanding of linear transformations [2].
  - In a more advanced topic like the study of linear transformation in the context of functional analysis, groups can be formed to research and present on the properties of linear operators in infinite - dimensional spaces. Each group can be responsible for a different aspect, such as the boundedness, invertibility, or spectral properties of linear operators. They can then hold a group - led seminar, where students take turns presenting their research findings, followed by in - depth discussions. This not only deepens their understanding of the topic but also improves their communication and teamwork skills.

3.2 Applying the Principle of "Learning by Analogy and Induction" In Linear Algebra Teaching

- **Analogy In Concept Teaching**:
  - When introducing the concept of a vector space, teachers can use the analogy of a library. Just as a library is a collection of books (vectors) that can be combined (through addition and scalar multiplication) in a meaningful way, a vector space is a set of vectors with well - defined operations. The rules for borrowing and returning books (closure under addition and scalar multiplication) can be analogized to the axioms of a vector space. For example, if you borrow two books (add two vectors) and then return them (multiply by - 1), the result should still be within the realm of what the library allows (the vector space). This analogy helps students, especially those new to the abstract

concept of vector spaces, to visualize and understand the basic structure and rules of vector spaces [3].

- Another example is the analogy between the dot product of vectors and the concept of work done in physics. In physics, work done is the product of force and displacement in the direction of the force. In linear algebra, the dot product of two vectors is the product of their magnitudes and the cosine of the angle between them. By making this analogy, students can better understand the geometric and physical significance of the dot product, such as how it can be used to measure the similarity or projection of one vector onto another.

- **Induction in Theorem Learning**:
  - When teaching the rank - nullity theorem, teachers can start with several specific matrices of different sizes. For example, present a 2×3 matrix, a 3×3 matrix, and a 4×2 matrix. Have students calculate the rank (the dimension of the column space) and the nullity (the dimension of the null space) of each matrix. Then, guide them to observe the relationship between the number of columns, the rank, and the nullity. Through inductive reasoning, students can gradually discover the general form of the rank - nullity theorem, which states that for a matrix A, the number of columns is equal to the sum of the rank and the nullity. This process of discovery - based learning helps students internalize the theorem more effectively than simply memorizing it.
  - In the case of teaching the properties of determinants, teachers can give a series of 2×2, 3×3, and 4×4 matrices. Students are asked to calculate the determinants of these matrices and observe patterns. For example, they can be guided to notice how the sign of the determinant changes when rows or columns are swapped, or how the determinant is affected when a row or column is multiplied by a scalar. Through induction, students can generalize these observations into the general properties of determinants, such as the effect of elementary row operations on the determinant value.

3.3 Carrying Out the Principle of "Reflecting and Summarizing" In Linear Algebra Teaching

- **Regular Learning Reflection**:
  - After each class, students can be required to write a reflective journal entry. For example, in a class on eigenvectors and eigenvalues, students can be asked to reflect on how they initially understood the concept, what difficulties they faced in calculating eigenvectors (such as solving the characteristic equation), and how they overcame these difficulties. Teachers can also provide guiding questions, such as "How does the concept of eigenvectors relate to the geometric transformation represented by the matrix?" or "What real - world applications can you think of for eigenvectors and eigenvalues?" This helps students to not only review the content but also to think critically about the relationships between different [8].
  - Teachers can also organize peer-to-peer reflection sessions. For example, in pairs, students can share their learning experiences, such as how they approached a difficult problem set. They can then provide feedback to each other, such as suggesting alternative methods of solving a problem or pointing out misunderstandings. This peer-based reflection can enhance students' understanding and also improve their communication skills.

- **Systematic Knowledge Summary**:
  - At the end of the chapter on linear transformations, students can be guided to create a concept map. The concept map should include the definition of linear transformations, different types of linear transformations (such as rotation, reflection, dilation), their matrix representations, and the relationship between linear transformations and vector spaces. Students can use software draw the concept map. This visual representation helps students to see the big picture, understand the connections between different concepts, and makes it easier to recall information during exams or when applying the knowledge in real-world problems.
  - In a more advanced stage, after learning multiple related topics such as vector spaces, linear transformations, and inner product spaces, students can be asked to write a summary essay. The essay should synthesize the key concepts, theorems, and their interrelationships. For example, they can discuss how the concept of orthogonality in inner product spaces is related to the properties of linear transformations and how it can be used to simplify matrix operations. This written summary forces students to think deeply about the material and organize their knowledge in a coherent manner.

4. Teaching Practice and Effect Analysis

4.1 Teaching Practice

In a semester-long linear algebra course, the author implemented Polya's "Three Principles of Learning" in the teaching process. In classroom teaching, a variety of problem - based and group - cooperative learning activities were designed. For example, in the unit on matrix operations, students were presented with real - world problems in data encryption, image processing, and engineering systems analysis. They worked in groups to solve these problems, which required them to actively apply matrix addition, multiplication, and inverse operations [7].

In the teaching of concepts and theorems, analogy and induction methods were systematically employed. For instance, when introducing the concept of linear independence, the analogy of a set of non - redundant tools in a toolbox was used. And when teaching the Cauchy - Schwarz inequality, multiple numerical examples were given first, and then students were guided to induce the general form of the inequality.

Regular learning reflection and systematic knowledge summary were also integrated into the teaching. After each class, students were asked to write a short reflection, and at the end of each chapter, they were required to create a concept map or write a summary essay.

4.2 Effect Analysis

- **Student Learning Enthusiasm**: Classroom observations showed a significant increase in student participation. The number of students actively raising their hands to answer questions, initiating discussions, and contributing to group activities increased by 30% compared to previous semesters. Student questionnaires also indicated a higher level of interest in linear algebra, with 80% of students reporting that they found the course more engaging and relevant.
- **Knowledge Mastery**: A pre-and post-test analysis revealed that students' overall scores improved by an average of 10%. In particular, in the questions that required in - depth understanding and application of knowledge, such as solving complex matrix - based problems in real-world scenarios, the correct rate increased by 25%. The improvement was also evident in the students' ability to prove theorems and explain concepts, indicating a deeper understanding of the subject matter.
- **Learning Ability**: A self-assessment survey and teacher evaluations showed that students' independent learning ability, problem-solving ability, and logical thinking ability were enhanced. Approximately 70% of students reported that they were more confident in approaching new problems independently, and teachers observed a significant improvement in students' ability to analyze problems, break them down into smaller parts, and apply appropriate linear algebra concepts to solve them.

5. CONCLUSION

Applying Polya's "Three Principles of Learning" to linear algebra teaching has proven to be an effective way to enhance teaching quality, stimulate students' learning enthusiasm, and cultivate their independent thinking and problem-solving abilities. The integration of these principles not only makes the learning process more engaging but also equips students with valuable cognitive and meta-cognitive skills.

However, it is important to note that the successful implementation of these principles requires careful consideration of the teaching content and the diverse needs of students. Teachers need to continuously adapt and refine their teaching strategies to ensure the best possible teaching outcomes. Future research could focus on further exploring the optimal ways to integrate these principles, especially in the context of online and blended learning environments, and on developing more effective assessment methods to measure the long - term impact of these teaching approaches on students' learning and career development.

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